**محاضرة ( 2 )**

Some General Concepts of Point Estimation :

We know that before data is available, the sample observations must be considered random variables X1, X2, ..., Xn. I It follows that any function of the Xi ’s - that is, any statistic - such as the sample mean X¯ or sample standard deviation S is also a random variable. Example: In the battery example just given, the estimator used to obtain the point estimate of µ was X¯ , and the point estimate of µ was 5.77. I If the three observed lifetimes had instead been x1 = 5.6, x2 = 4.5, and x3 = 6.1, use of the estimator X¯ would have resulted in the estimate x¯ = (5.6 + 4.5 + 6.1)/3 = 5.40. I The symbol θˆ (“theta hat”) is customarily used to denote both the estimator of µ and the point estimate resulting from a given sample.

Thus θ ˆ = X¯ is read as “the point estimator of θ is the sample mean X¯ .” The statement “the point estimate of θ is 5.77” can be written concisely as θˆ = 5.77. In the best of all possible worlds, we could find an estimator θˆ for which θˆ = θ always. However, θˆ is a function of the sample Xi ’s, so it is a random variable. I For some samples, θˆ will yield a value larger than θ, whereas for other samples θˆ will underestimate θ. If we write θ ˆ = θ + error of estimation then an accurate estimator would be one resulting in small estimation errors, so that estimated values will be near the true value.

A sensible way to quantify the idea of θˆ being close to θ is to consider the squared error (θˆ− θ) 2 . For some samples, θˆ will be quite close to θ and the resulting squared error will be near 0. I Other samples may give values of θˆ far from θ, corresponding to very large squared errors. A measure of accuracy is the expected or mean square error (MSE) MSE = E[(θ ˆ− θ) 2 ] I If a first estimator has smaller MSE than does a second, it is natural to say that the first estimator is the better one.

However, MSE will generally depend on the value of θ. What often happens is that one estimator will have a smaller MSE for some values of θ and a larger MSE for other values. I Finding an estimator with the smallest MSE is typically not possible. One way out of this dilemma is to restrict attention just to estimators that have some specified desirable property and then find the best estimator in this restricted group. I A popular property of this sort in the statistical community is unbiasedness .

**properties of good estimation**

**1- Unbiasedness**

Definition : A point estimator θˆ is said to be an unbiased estimator of θ if E(θˆ) = θ for every possible value of θ. If θ ˆ is not unbiased, the difference E(θ ˆ− θ) is called the bias of θˆ.

That is, θˆ is unbiased if its probability (i.e., sampling) distribution is always centered at the true value of the parameter.

Example : The sample proportion X/n can be used as an estimator of p, where X, the number of sample successes, has a binomial distribution with parameters n and p.Thus E(ˆp) = E( X n ) = 1 n E(X) = 1 n (np) = p , Proposition When X is a binomial rv with parameters n and p, the sample proportion pˆ = X/n is an unbiased estimator of p. I No matter what the true value of p is, the distribution of the estimator will be centered at the true value.